

Short communication

Simplified expressions for radiation scattering in canopies with ellipsoidal leaf angle distributions

 G.N. Flerchinger^{a,*}, Qiang Yu^b
^a USDA Agricultural Research Service, Northwest Watershed Research Center, 800 Park Boulevard, Suite 105, Boise, ID 83712, USA

^b Institute of Geographical Sciences and Natural Resources Research, Chinese Academy of Sciences, Beijing, PR China

Received 19 December 2006; received in revised form 5 March 2007; accepted 10 March 2007

Abstract

The ability to simulate the surface energy balance and microclimate within a plant canopy is contingent upon accurate simulation of radiation exchange within the canopy. Accurate radiation simulations require some assumption of leaf angle distribution to compute transmissivity, reflection and scattering of radiation. The ellipsoidal leaf angle density function can very closely approximate real plant canopies but requires complex integrations for different combinations of leaf area index, incident radiation angle, and density function. This paper presents close approximations ($R^2 > 0.99$) to compute the transmissivity and scattering functions for elliptical leaf angle distributions that can be more easily implemented into simulation models.

© 2007 Elsevier B.V. All rights reserved.

Keywords: Long-wave radiation; Multi-layer canopy; Multi-source canopy; PAR; Short-wave radiation; Surface energy balance

1. Introduction

Radiation exchange within the canopy plays a crucial role in the canopy microclimate. This exchange is the driving force for the surface energy balance, influences canopy leaf temperature, and provides the energy for photosynthesis and plant growth. Accurate simulation of canopy microclimate is contingent on simulation of the surface radiation balance.

Estimation of the surface radiation balance differs widely between simulation models depending on whether they are single-layer (Monteith, 1963), dual-source (Shuttleworth and Wallace, 1985; Huntingford et al., 1995), or multiple-layer models (Norman, 1979; Smith and Goltz, 1994; Stockle, 1990; Flerchinger et al., 1998; Zhao and Qualls, 2005). Multiple-layer models simulate radiation distribution and microcli-

mate profiles throughout the canopy, requiring estimation of the transmissivity, reflection, and scattering of radiation within each canopy layer, which invariably requires assumptions regarding leaf orientation.

Multiple-layer canopy models must compute the energy balance for each layer within the canopy. This requires computation for downward direct, and upward and downward diffuse radiation being transmitted, reflected, scattered, and absorbed by each layer. Upward flux of diffuse short-wave radiation between canopy layer i and the next layer below, $i + 1$, ($S_{u,i}$) can be estimated as:

$$\begin{aligned}
 S_{u,i} = & \tau_{d,i+1} S_{u,i+1} + (\beta_1 f_{d,\downarrow\uparrow} + \tau_1 f_{d,\downarrow\downarrow}) \\
 & \times (1 - \tau_{d,i+1}) S_{d,i} + (\beta_1 f_{d,\uparrow\uparrow} + \tau_1 f_{d,\uparrow\downarrow}) \\
 & \times (1 - \tau_{d,i}) S_{u,i+1} + \beta_1 f_{b,\downarrow\uparrow} (1 - \tau_{b,i+1}) S_{b,i},
 \end{aligned}
 \tag{1}$$

where $\tau_{d,i}$ is the transmissivity of canopy layer i to diffuse radiation (i.e. the fraction of diffuse radiation going through the canopy layer unimpeded), $\tau_{b,i+1}$ the

* Corresponding author. Tel.: +1 208 422 0716; fax: +1 208 334 1502.

E-mail address: gflerchi@nwrc.ars.usda.gov (G.N. Flerchinger).

transmissivity of canopy layer $i + 1$ to direct (or beam) radiation, β_l the albedo of the canopy leaves, τ_l the leaf transmissivity, $f_{d,\uparrow}$ the fraction of reflected downward diffuse radiation that is scattered upward, $f_{d,\downarrow}$ the fraction of reflected downward diffuse radiation that is scattered downward, $f_{b,\uparrow}$ the fraction of reflected downward direct radiation that is scattered upward, and $S_{b,i}$ is the direct radiation entering canopy layer $i + 1$. It should be noted that $f_{d,\uparrow} = f_{d,\downarrow} = 1 - f_{d,\downarrow} = 1 - f_{d,\uparrow}$. Conversely, the fraction of downward diffuse radiation transmitted through the leaves that is scattered upward is equal to $1 - f_{d,\uparrow}$ and that scattered downward is $1 - f_{d,\downarrow}$. These same equalities hold true for downward direct radiation.

An expression similar to Eq. (1) can be written for downward radiation at any point in the canopy. Eq. (1) can be applied for any portion of the short-wave spectrum provided appropriate values for β_l and τ_l are used. Long-wave radiation can be computed similarly by including emitted long-wave radiation within the layer and ignoring direct radiation.

Common simplifications for leaf angle distribution are to assume the leaves are horizontally, vertically or spherically distributed (Flerchinger, 2000; Zhao and Qualls, 2005). A more generalized approach is to assume an ellipsoidal leaf angle density function, as presented by Campbell (1986) and Campbell and Norman (1998). The ellipsoidal distribution gives good approximation to leaf angle distribution of real canopies (Barclay, 2001; Falster and Westoby, 2003) and can be extended to account for clumping of the leaves. However, the ellipsoidal distribution has had limited use, in part because of the complex computations required for scattering of reflected radiation and transmissivity of diffuse radiation. Computations for $f_{b,\uparrow}$, $f_{d,\uparrow}$, and τ_d in Eq. (1) require integration of complex functions for different combinations of leaf area index, leaf angle distribution, and incident radiation angle, and therefore must be re-computed at various time-steps within the model. The objective of this paper is to develop very close approximations to these relations that can be more easily implemented into multi-layer canopy models.

2. Theory

2.1. Transmission of radiation through the canopy

The transmissivity of a canopy layer with a leaf area index of L to direct radiation is calculated from:

$$\tau_b = \exp(-K_b L), \quad (2)$$

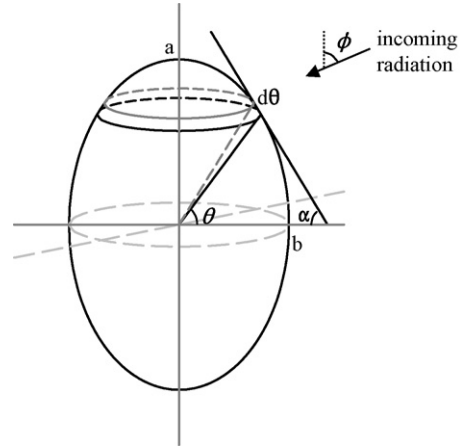


Fig. 1. Variables and geometric relations for an ellipsoid. Leaf angle is α , orientation of the leaf angle upon the ellipsoid is defined by θ , and the zenith angle of incoming radiation is ϕ . Axes of the ellipsoid are a and b .

where τ_b is the fraction direct radiation passing through a canopy layer unimpeded by the vegetation, and K_b is an extinction coefficient for direct radiation, which is dependent on the direction of the radiation source and the orientation of the plant leaves. Campbell and Norman (1998) present an expression for K_b assuming an ellipsoidal leaf orientation:

$$K_b = \frac{\sqrt{x^2 + \tan^2 \phi}}{x + 1.774(x + 1.182)^{-0.733}}, \quad (3)$$

where x is a coefficient relating to leaf orientation and ϕ is the zenith angle of the radiation (Fig. 1). The value of x is related to the vertical (a) and horizontal (b) axes of the ellipsoid shown in Fig. 1 by $x = b/a$. For vertical leaf elements, $x = 0$; for spherically oriented elements, $x = 1$; for horizontal elements, $x = \infty$. Typical values of x for different crops are given by Campbell and Norman (1998).

Diffuse radiation comes from all directions. The transmission of diffuse radiation from a given direction is identical to that for direct radiation for that direction. Thus, the transmission of diffuse radiation through the canopy can be calculated by integrating the expression for direct radiation over all directions within the hemisphere (Campbell and Norman, 1998):

$$\tau_d = 2 \int_0^{\pi/2} \tau_b(\phi) \sin \phi \cos \phi d\phi, \quad (4)$$

where τ_d is the fraction of diffuse radiation passing through a canopy layer unimpeded by vegetation. This expression requires numerical integration upon substitution of Eqs. (2) and (3) for τ_b .

2.2. Scattering of radiation within the canopy

A portion of the direct radiation reflected from a leaf surface is scattered upward; the remainder is scattered downward. The fraction of reflected direct radiation scattered upward for spherically oriented leaves is (Zhao and Qualls, 2005):

$$f_{b,\downarrow\uparrow} = \int_{\omega=0}^{2\pi} \int_{\theta=-\pi/2}^{\pi/2} \left| \sin \theta \cos \phi + \cos \theta \sin \phi \cos \omega \right| f_u(\alpha) \cos \theta d\theta d\omega, \quad (5)$$

where θ is the angle from the horizontal, ω the azimuth angle, and $f_u(\alpha)$ is the upward fraction of reflected radiation for a leaf inclination angle of α . The function $f_u(\alpha)$ is given by (Norman and Jarvis, 1975):

$$f_u(\alpha) = \frac{1}{2}(1 + \cos \alpha) = \cos^2 \frac{\alpha}{2}. \quad (6)$$

Leaf angle, α , can be computed from its orientation on the ellipsoid, θ (see Fig. 1) from:

$$\tan(\alpha) = \frac{\tan((\pi/2) - \theta)}{x^2}. \quad (7)$$

Eq. (5) can be generalized to ellipsoidal distributions by incorporating a leaf angle distribution function presented by Campbell (1990). Numerical integration of Eqs. (5)–(7) is relatively straightforward for spherically oriented leaves, but considerably more complicated for an ellipsoidal distribution. Norman and Jarvis (1975) present an alternative approach to compute upward scattering applicable for any leaf angle distribution. This is done by weighting the scattering function $f_u(\alpha)$ for a given leaf angle by the fraction of leaf area projected in the direction of the incoming beam radiation.

Eq. (6) assumes all radiation is incident on the top of the leaves. Depending on the zenith angle (ϕ), much of the radiation strikes the bottom of the leaves. The fraction of radiation incident on the tops of leaves is given by (Norman and Jarvis, 1975):

$$f_{\text{top}} = \frac{\psi}{2\psi + \cos \alpha \cos \phi}, \quad (8)$$

where ψ is defined as:

$$\psi = \frac{\sin \alpha \sin \phi \sin \varepsilon - \varepsilon \cos \alpha \cos \phi}{\pi}. \quad (9)$$

Here, ε is defined as:

$$\cos \varepsilon = \frac{1}{\tan \alpha \tan \phi}. \quad (10)$$

Assuming that the reflective properties for the top and bottom of the leaves are identical and recognizing that the fraction of reflected radiation incident on the bottom of the leaves that is scattered upward is $1 - f_u(\alpha)$, the fraction of reflected radiation incident at angle ϕ scattered upward from leaves oriented at angle α is:

$$f_u(\alpha, \phi) = \frac{\psi}{2\psi + \cos \alpha \cos \phi} f_u(\alpha) + \left[1 - \frac{\psi}{2\psi + \cos \alpha \cos \phi} \right] (1 - f_u(\alpha)). \quad (11)$$

The fraction of leaves inclined at angle α projected in the direction of a radiation beam at zenith angle ϕ is given by:

$$g(\alpha, \phi) = \cos \alpha \cos \phi, \quad \text{for } (\alpha + \phi) \leq 90^\circ, \\ g(\alpha, \phi) = 2\psi + \cos \alpha \cos \phi, \quad \text{for } (\alpha + \phi) > 90^\circ. \quad (12)$$

Norman and Jarvis (1975) present tables for $f_u(\alpha, \phi)$ computed from Eq. (11) and weighting functions from Eq. (12) for various combinations of α and ϕ .

The fraction of reflected radiation scattered upward for any leaf inclination distribution can be computed by integrating $f_u(\alpha, \phi)$ across leaf angles, weighted by Eq. (12) and the fraction of leaves at each angle increment. For an ellipsoidal distribution:

$$f_{b,\downarrow\uparrow} = \frac{\int_{\theta=0}^{\pi/2} f_u(\alpha, \phi) g(\alpha, \phi) dA d\theta}{\int_{\theta=0}^{\pi/2} g(\alpha, \phi) dA d\theta}, \quad (13)$$

where dA is the incremental distribution of leaves within the angle increment $d\theta$. For an ellipsoid, this is the portion of surface area of the ellipsoid within the angle increment $\theta + d\theta$ (Fig. 1).

The fraction of reflected downward diffuse radiation scattered upward is computed by integrating that for direct radiation over all incident radiation angles, weighted by the portion of radiation at each incident angle:

$$f_{d,\downarrow\uparrow} = \int_{\phi=0}^{\pi/2} f_{b,\downarrow\uparrow} \sin \phi d\phi. \quad (14)$$

3. Methods

Transmission of diffuse radiation, τ_d , was computed for values of L of 0.01 and from 0.2 to 8.0 in increments

of 0.2 and for values of x ranging from 0 to 4.0 in increments of 0.2 as well as $x = 100$. Numerical integrations of Eq. (4) were performed using trapezoidal approximations. Iterations were performed by doubling the number of integration elements until an error tolerance of 0.00001 in transmissivity was reached. The number of elements required to satisfy the error tolerance was typically 512, but ranged as high as 2048.

The fraction of reflected direct radiation scattered upward, $f_{b,\uparrow}$, was computed for the same values of x mentioned previously and for zenith angles ranging from 0° to 90° in increments of 1° by numerically integrating Eq. (13). Numerical integrations were performed using rectangular approximations computed from values at the midpoint of the integration elements. The surface area of the ellipsoid within the angle increment $\theta + d\theta$ (Fig. 1) was computed from the length of the secant on the ellipse between $\theta + d\theta$ multiplied by the average circumference of the ellipsoid between $\theta + d\theta$. Iterations were performed by doubling the number of integration elements until an error tolerance of 0.00001 in $f_{b,\uparrow}$ was obtained. A minimum of 128 elements was used for all integrations, which was often sufficient for lower values of ϕ and x . For x above 3.0 at zenith angles approaching the horizon, typically as many 1024 element were required. Convergence was difficult for $x = 100$ at zenith angles approaching the horizon, requiring as many as 10^8 increments (Tolerance could have been attained with fewer elements if finer elements were used for values of θ near zero and coarser elements elsewhere.).

The fraction of reflected downward diffuse radiation scattered upward, $f_{d,\uparrow}$, was computed for the same values of x mentioned previously by numerically integrating Eq. (14). Numerical integrations were performed using trapezoidal approximations and the values of $f_{b,\uparrow}$ computed previously in 1° increments of zenith angle.

The relations resulting from all numerical integrations were plotted to discern the equation form to best fit the data. Non-linear least squares regressions were performed using the NLIN procedure within the SAS software package (SAS Institute, Cary, NC, U.S.A.).

4. Results

The relation between the extinction coefficient for diffuse radiation (K_d) and L obtained from Eqs. (2)–(4) for various values of x is presented in Fig. 2, which is similar to Fig 15.4 presented by Campbell and Norman (1998). From Fig. 2, K_d decreases with L and approaches a different horizontal asymptote

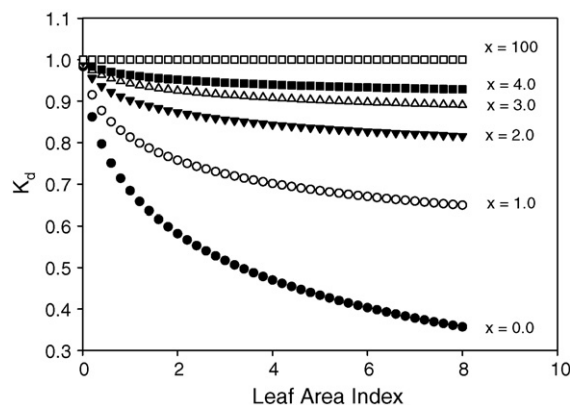


Fig. 2. Relation between the extinction coefficient for diffuse radiation (K_d) and leaf area index (L) obtained from integrating Eqs. (2)–(4) for various ellipsoidal distributions (x).

depending on the value of x . A typical function to describe a horizontal asymptote (Riddle, 1979) has the form:

$$K_d = \frac{K_{d\infty}L^A + B}{L^A + B}, \quad (15)$$

where A and B are empirical coefficients, and $K_{d\infty}$ is the asymptote that K_d approaches at infinite L for a given value of x . Integrations of Eq. (4) with a leaf area index of 600 gives an approximate relation of $K_{d\infty}$ for various values x , which is plotted in Fig. 3. Values of $K_{d\infty}$ range from near zero for $x=0$ and approach 1.0 as x approaches infinity. This relation can be approximated by:

$$K_{d\infty} = \frac{2}{\pi} \arctan(x), \quad (16)$$

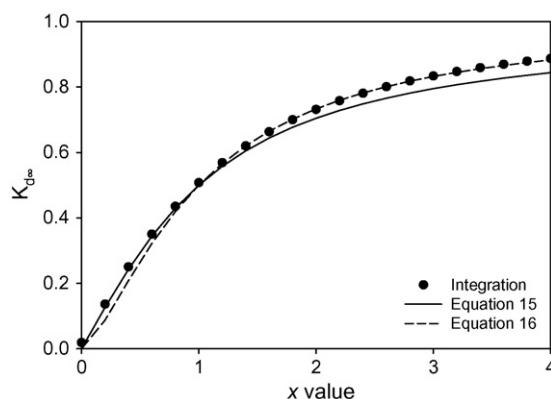


Fig. 3. Approximate relation between extinction coefficient for diffuse radiation with infinite leaf area index ($K_{d\infty}$) and varying ellipsoidal distributions (x).

or, alternatively by the horizontal asymptote function:

$$K_{d\infty} = \frac{x^C}{x^C + 1.0}, \quad (17)$$

where C is an empirical coefficient. A non-linear least-squares regression for K_d for all values x and L was performed upon combining Eqs. (15) and (16) and a separate regression was performed for the combination of Eqs. (15) and (17). These resulted in values for A , B and C of 0.65, 1.9, and 1.46, respectively. The best-fit values for A and B were independent of whether Eq. (16) or (17) was used. Implementing Eq. (16) yielded a coefficient of determination for K_d of 0.985 while Eq. (17) yielded 0.995. The plot of Fig. 3 suggests that Eq. (16) is more suitable for $x < 1$ while Eq. (17) is better for $x > 1$. However, the fraction of variation in τ_d accounted for by using either method was 0.9999. The maximum error in τ_d when using a combination of Eqs. (16) and (17) ranged from -0.023 for x of 0.0 and leaf index of 3.0 to $+0.011$ for x of 0.6 and leaf area index of 2.0.

Because Eqs. (2)–(4) assume leaves are distributed randomly within the canopy space, Eqs. (15)–(17) also employ this assumption. However, if leaves are clumped, such as with canopy gaps or row crops, canopy transmission can be approximated by multiplying leaf area in Eq. (2) by a clumping factor to account for the fact that the leaves are less efficient at intercepting radiation (Campbell and Norman, 1998). The same clumping factor can be applied to the exponential decay function for diffuse radiation. In this case, leaf area in Eq. (15) should also be multiplied by the clumping factor when computing the extinction coefficient K_d .

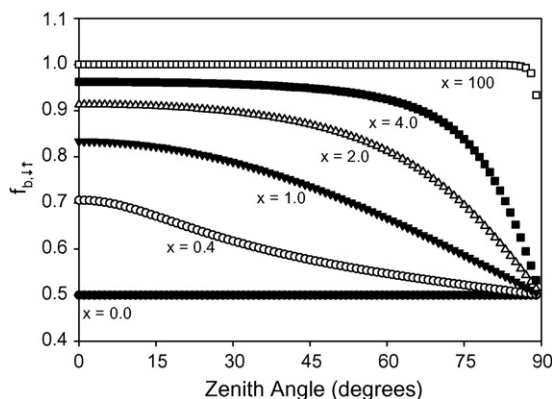


Fig. 4. The fraction of reflected downward direct radiation scattered upward ($f_{b,\downarrow\uparrow}$) for varying zenith angles (ϕ) and ellipsoidal distributions (x).

The relation between $f_{b,\downarrow\uparrow}$ and ϕ is plotted in Fig. 4 for various values of x . A very close approximation ($R^2 > 0.9999$) for $f_{b,\downarrow\uparrow}$ for spherical leaf orientation presented in Zhao and Qualls (2005) is:

$$f_{b,\downarrow\uparrow} = 0.5 + 0.3334 \cos \phi. \quad (18)$$

This expression can be generalized to all ellipsoidal leaf orientations as follows:

$$f_{b,\downarrow\uparrow} = 0.5 + 0.5 \left(\frac{2}{\pi} \arctan(x) \right)^D x^{(\sin \phi)^{E+Fx}} \cos \phi. \quad (19)$$

The form of Eq. (19) forces an exact fit for horizontal leaf orientations. The arctangent expression in Eq. (19) was derived by recognizing from Fig. 4 that $f_{b,\downarrow\uparrow}$ varies from 0.5 at $x = 0$ to 1.0 at $x = \infty$ for zero zenith angles. The equation can be forced to match the expression presented by Zhao and Qualls (2005) for a spherical leaf orientation ($x = 1$) by setting D to 0.585. The exponential term for x was selected (after numerous trials) by graphing the ratio of $f_{b,\downarrow\uparrow}$ at $\phi = 0.0$ to $f_{b,\downarrow\uparrow}$ at other zenith angles for various values of x . A non-linear least-squares regression (omitting the data for $x = 100$) yielded values for E and F of 0.569 and 1.09 with a coefficient of determination for $f_{b,\downarrow\uparrow}$ of 0.991. The maximum error in $f_{b,\downarrow\uparrow}$ occurs at an x -value of 4.0 and ranges from -0.05 at a zenith angle of 35° to $+0.05$ at 70° . Eq. (19) yields values greater than 1.0 for values of x greater than 5.7, which is clearly beyond the range of applicability for this equation. Above $x = 5.0$, leaf orientation can be considered horizontal and $f_{b,\downarrow\uparrow}$ set to 1.0 with very little error except for zenith angles near horizontal.

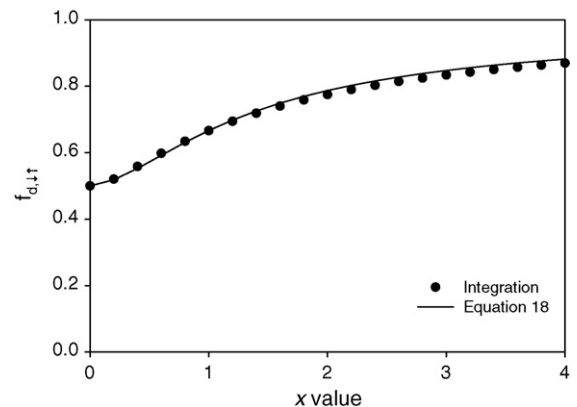


Fig. 5. The fraction of reflected downward diffuse radiation scattered upward ($f_{d,\downarrow\uparrow}$) for varying ellipsoidal distributions (x).

The relation between the fraction of reflected downward diffuse radiation scattered upward ($f_{d,\downarrow\uparrow}$) and x is plotted in Fig. 5. Like $f_{b,\downarrow\uparrow}$, the relation varies from 0.5 for $x=0.0$ to 1.0 for $x=\infty$. A similar expression can be used to approximate $f_{d,\downarrow\uparrow}$:

$$f_{d,\downarrow\uparrow} = 0.5 + 0.5 \left(\frac{2}{\pi} \arctan(x) \right)^G. \quad (20)$$

The value of $f_{d,\downarrow\uparrow}$ for spherical leaf orientation is $2/3$, from which the exponent G can be back-calculated to be 1.585. This yields a coefficient of determination for $f_{d,\downarrow\uparrow}$ of 0.999 with a maximum error of 0.013 at an x -value of 3.0.

5. Conclusions

Expressions for transmissivity of diffuse radiation and scattering of direct and diffuse radiation were integrated for a range of ellipsoidal leaf angle distributions, leaf area indices, and incident solar angles. Simplified algorithms were developed which very closely approximate these relations, with coefficient of determinations exceeding 0.99. Extension of the transmissivity equations for clumped vegetation was discussed; scattering equations are independent of vegetation clumping. The simplified algorithms can be more easily implemented in simulation models of radiation transfer within the canopy and will be more computationally efficient than numerical integrations requiring hundreds or even thousands of integration elements.

References

- Barclay, H., 2001. Distribution of leaf orientations in six conifer species. *Can. J. Bot.* 79, 389–403.
- Campbell, G.S., 1986. Extinction coefficients for radiation in plant canopies calculated using an ellipsoidal inclination angle distribution. *Agric. Forest Meteorol.* 36, 317–321.
- Campbell, G.S., 1990. Derivation of an angle density function for canopies with ellipsoidal leaf angle distribution. *Agric. Forest Meteorol.* 49, 173–176.
- Campbell, G.S., Norman, J.H., 1998. *An Introduction to Environmental Biophysics*, 2nd ed. Springer, New York, 186 pp.
- Falster, D., Westoby, M., 2003. Leaf size and angle vary widely across species: what consequences for light interception? *New Phytol.* 158, 509–525.
- Flerchinger, G.N., 2000. The simultaneous heat and water (SHAW) model: technical documentation. Technical Report NWRC-2000-09, USDA-ARS, Northwest Watershed Research Center, Boise, Idaho (Electronic document: <http://www.nwrc.ars.usda.gov/models/SHAW/downloads.html>).
- Flerchinger, G.N., Kustas, W.P., Weltz, M.A., 1998. Simulating surface energy fluxes and radiometric surface temperatures for two arid vegetation communities using the SHAW model. *J. Appl. Meteorol.* 37 (5), 449–460.
- Huntingford, C., Allen, S.J., Harding, R.J., 1995. An intercomparison of single and dual-source vegetation-atmosphere transfer models applied to transpiration from Sahelian savannah. *Boundary-Layer Meteorol.* 74 (4), 397–418.
- Monteith, J.L., 1963. Gas exchange in plant communities. In: Evans, L.T. (Ed.), *Environmental Control of Plant Growth*. Springer, New York, pp. 95–112.
- Norman, J.M., 1979. Modeling the complete crop canopy. In: Barfield, B.J., Gerber, J.F. (Eds.), *Modification of the Aerial Environment of Plants*. American Society of Agricultural Engineers, St. Joseph, MI, pp. 249–277.
- Norman, J.M., Jarvis, P.G., 1975. Photosynthesis in Sitka spruce (*Picea sitchensis* (Bong) Carr.). V. Radiation penetration theory and a test case. *J. Appl. Ecol.* 12, 839–878.
- Riddle, D.F., 1979. *Calculus and Analytical Geometry*, 3rd ed. Wadsworth Publishing Company Inc., Belmont, CA, 883 pp.
- Shuttleworth, W.J., Wallace, J.S., 1985. Evaporation from sparse crops—an energy combination theory. *Quart. J. Roy. Meteorol. Soc.* 111, 839–855.
- Smith, J.A., Goltz, S.M., 1994. A thermal exitance and energy balance model for forest canopies. *IEEE Trans. Geosci. Remote Sens.* 32, 1160–1166.
- Stockle, C.O., 1990. Canopy photosynthesis and transpiration estimates using radiation interception models with different levels of detail. *Ecol. Model.* 60, 31–44.
- Zhao, W., Qualls, R.J., 2005. A multiple-layer canopy scattering model to simulate shortwave radiation distribution within a homogeneous plant canopy. *Water Resour. Res.* 41, doi:10.1029/2005WR004016 W08409.